

# Ramsey numbers and adiabatic quantum computing

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(Dated: January 14, 2013)

The graph-theoretic Ramsey numbers are notoriously difficult to calculate. In fact, for the two-color Ramsey numbers  $R(m, n)$  with  $m, n \geq 3$ , only nine are currently known. We present a quantum algorithm for the computation of the Ramsey numbers  $R(m, n)$ . We show how the computation of  $R(m, n)$  can be mapped to a combinatorial optimization problem whose solution can be found using adiabatic quantum evolution. We numerically simulate this adiabatic quantum algorithm and show that it correctly determines the Ramsey numbers  $R(3, 3)$  and  $R(2, s)$  for  $5 \leq s \leq 7$ . We then discuss the algorithm's experimental implementation, and close by showing that Ramsey number computation belongs to the quantum complexity class QMA.

PACS numbers: 03.67.Ac, 02.10.Ox, 89.75.Hc

In an arbitrary party of  $N$  people one might ask whether there is a group of  $m$  people who are all mutually acquainted, or a group of  $n$  people who are all mutual strangers. Using Ramsey theory [1, 2], it can be shown that a threshold value  $R(m, n)$  exists for the party size  $N$  so that when  $N \geq R(m, n)$ , all parties of  $N$  people will either contain  $m$  mutual acquaintances, or  $n$  mutual strangers. The threshold value  $R(m, n)$  is an example of a two-color Ramsey number. Other types of Ramsey numbers exist, though we will focus on two color Ramsey numbers in this paper.

One can represent the  $N$ -person party problem by an  $N$ -vertex graph. Here each person is associated with a vertex, and an edge is drawn between a pair of vertices only when the corresponding people know each other. In the case where  $m$  people are mutual acquaintances, there will be an edge connecting any pair of the  $m$  corresponding vertices. Similarly, if  $n$  people are mutual strangers, there will be no edge between any of the  $n$  corresponding vertices. In the language of graph theory [3], the  $m$  vertices form an  $m$ -clique, and the  $n$  vertices form an  $n$ -independent set. The party problem is now a statement in graph theory: if  $N \geq R(m, n)$ , every graph with  $N$  vertices will contain either an  $m$ -clique, or an  $n$ -independent set. Ramsey numbers can also be introduced using colorings of complete graphs, and  $R(m, n)$  corresponds to the case where only two colors are used.

Ramsey theory has found applications in mathematics, information theory, and theoretical computer science [6]. An application of fundamental significance appears in the Paris-Harrington (PH) theorem of mathematical logic [4] which established that a particular statement in Ramsey theory related to graph colorings and natural numbers is true, though unprovable within the axioms of Peano arithmetic. Such statements are known to exist as a consequence of Godel's incompleteness theorem, though the PH theorem provided the first natural example. Deep connections have also been shown to exist between Ramsey theory, topological dynamics, and ergodic theory [5].

Ramsey numbers grow extremely quickly and so are notoriously difficult to calculate. In fact, for two color Ramsey numbers  $R(m, n)$  with  $m, n \geq 3$ , only nine are presently known[3]. To check whether  $N \stackrel{?}{=} R(m, n)$  requires examining *all*  $2^{N(N-1)/2}$   $N$ -vertex graphs. The number of graphs to be checked thus grows super-exponentially with  $N$ , and so the task quickly becomes intractable. Ketonen and Solovay [7] have shown that this is the root cause for why the statement in the PH theorem cannot be proved within Peano arithmetic.

In this paper we: (i) present a quantum algorithm for calculating Ramsey numbers based on adiabatic quantum evolution; (ii) numerically simulate the algorithm to verify that it correctly calculates small Ramsey numbers; (iii) discuss its experimental implementation; and (iv) show that Ramsey number computation belongs to the quantum complexity class QMA.

**Optimization Problem:** We begin by establishing a 1-1 correspondence between the set of  $N$ -vertex graphs and binary strings of length  $L = N(N-1)/2$ . To each  $N$ -vertex graph  $G$  there corresponds a unique adjacency matrix  $A(G)$  which is an  $N \times N$  symmetric matrix with vanishing diagonal matrix elements, and with off-diagonal element  $a_{i,j} = 1$  (0) when distinct vertices  $i$  and  $j$  are (are not) joined by an edge. It follows that  $A(G)$  is determined by its lower triangular part. By concatenating column-wise the matrix elements  $a_{i,j}$  appearing below the principal diagonal, we can construct a unique binary string  $g(G)$  of length  $L$  for each graph  $G$ :

$$g(G) \equiv a_{2,1} \cdots a_{N,1} a_{3,2} \cdots a_{N,2} \cdots a_{N,N-1}. \quad (1)$$

Given the string  $g(G)$ , the following procedure determines the number of  $m$ -cliques in  $G$ . Choose  $m$  vertices  $S_\alpha = \{v_1, \dots, v_m\}$  from the  $N$  vertices of  $G$  and form the product  $\mathcal{C}_\alpha = \prod_{(j \neq k)}^{(v_j, v_k \in S_\alpha)} a_{v_j, v_k}$ . Note that  $\mathcal{C}_\alpha = 1$  when  $S_\alpha$  forms an  $m$ -clique; otherwise  $\mathcal{C}_\alpha = 0$ . Now repeat this procedure for all  $\rho = C(N, m)$  ways of choosing  $m$  vertices from  $N$  vertices, and form the sum  $\mathcal{C}(G) =$

$\sum_{\alpha=1}^{\rho} \mathcal{C}_{\alpha}$ . By construction,  $\mathcal{C}(G)$  equals the number of  $m$ -cliques contained in  $G$ . A similar procedure determines the number of  $n$ -independent sets in  $G$ . Briefly, choose  $n$  vertices  $T_{\alpha} = \{v_1, \dots, v_n\}$  from the  $N$  vertices in  $G$ , and form the product  $\mathcal{I}_{\alpha} = \prod_{(j \neq k) \atop (v_j, v_k \in T_{\alpha})} \bar{a}_{v_j, v_k}$ , where  $\bar{a}_{v_j, v_k} = 1 - a_{v_j, v_k}$ . If the vertex set  $T_{\alpha}$  forms an  $n$ -independent set, then  $\mathcal{I}_{\alpha} = 1$ ; otherwise  $\mathcal{I}_{\alpha} = 0$ . Repeat this for all  $\nu = C(N, n)$  ways of choosing  $n$  vertices from  $N$  vertices, then form the sum  $\mathcal{I}(G) = \sum_{\alpha=1}^{\nu} \mathcal{I}_{\alpha}$ . By construction,  $\mathcal{I}(G)$  gives the number of  $n$ -independent sets contained in  $G$ . Finally, define

$$h(G) = \mathcal{C}(G) + \mathcal{I}(G). \quad (2)$$

It follows from the above discussion that  $h(G)$  is the total number of  $m$ -cliques and  $n$ -independent sets in  $G$ . Thus  $h(G) \geq 0$  for all graphs  $G$ ; and  $h(G) = 0$  if and only if  $G$  does not contain an  $m$ -clique or  $n$ -independent set.

We can use  $h(G)$  as the cost function for the following combinatorial optimization problem. For given integers  $(N, m, n)$ , and with  $h(G)$  defined as above, find an  $N$ -vertex graph  $G_*$  that yields the global minimum of  $h(G)$ . Notice that if  $N < R(m, n)$ , the (global) minimum will be  $h(G_*) = 0$  since Ramsey theory guarantees that a graph exists which has no  $m$ -clique or  $n$ -independent set. On the other hand, if  $N \geq R(m, n)$ , Ramsey theory guarantees  $h(G_*) > 0$ . If we begin with  $N < R(m, n)$  and increment  $N$  by 1 until we first find  $h(G_*) > 0$ , then the corresponding  $N$  will be exactly  $R(m, n)$ . We now show how this combinatorial optimization problem can be solved using adiabatic quantum evolution.

**Quantum Algorithm:** The adiabatic quantum evolution (AQE) algorithm [8] exploits the adiabatic dynamics of a quantum system to solve combinatorial optimization problems. The AQE algorithm uses the optimization problem cost function to define a problem Hamiltonian  $H_P$  whose ground-state subspace encodes all problem solutions. The algorithm evolves the state of an  $L$ -qubit register from the ground-state of an initial Hamiltonian  $H_i$  to the ground-state of  $H_P$  with probability approaching 1 in the adiabatic limit. An appropriate measurement at the end of the adiabatic evolution yields a solution of the optimization problem almost certainly. The time-dependent Hamiltonian  $H(t)$  for global AQE is

$$H(t) = \left(1 - \frac{t}{T}\right) H_i + \left(\frac{t}{T}\right) H_P, \quad (3)$$

where  $T$  is the algorithm runtime, and adiabatic dynamics corresponds to  $T \rightarrow \infty$ .

To map the optimization problem associated with computing  $R(m, n)$  onto an adiabatic quantum computation, we begin with the 1-1 correspondence between  $N$ -vertex graphs  $G$  and length  $L = N(N-1)/2$  binary strings  $g(G)$ . From Eq. (1) we see that position along the string is indexed by vertex pairs  $(i, j)$ . We thus identify a qubit with each such pair  $(i, j)$ , and will thus need  $L$  qubits.

Defining the computational basis states (CBS) to be the eigenstates of  $\sigma_z^0 \otimes \dots \otimes \sigma_z^{L-1}$ , we identify the  $2^L$  graph strings  $g(G)$  with the  $2^L$  CBS:  $g(G) \rightarrow |g(G)\rangle$ . The problem Hamiltonian  $H_P$  is defined to be diagonal in the computational basis with eigenvalue  $h(G)$  associated with eigenstate  $|g(G)\rangle$ :

$$H_P |g(G)\rangle = h(G) |g(G)\rangle. \quad (4)$$

Note that the ground-state energy of  $H_P$  will be zero iff there is a graph with no  $m$ -cliques or  $n$ -independent sets. We give an operator expression for  $H_P$  below. The initial Hamiltonian  $H_i$  is chosen to be

$$H_i = \sum_{l=0}^{L-1} \frac{1}{2} (I^l - \sigma_x^l), \quad (5)$$

where  $I^l$  and  $\sigma_x^l$  are the identity and x-Pauli operator for qubit  $l$ , respectively. The ground-state of  $H_i$  is the easily constructed uniform superposition of CBS.

The quantum algorithm for computing  $R(m, n)$  begins by setting  $N$  equal to a strict lower bound for  $R(m, n)$  which can be found using the probabilistic method [9] or a table of two-color Ramsey numbers [3]. The AQE algorithm is run on  $L_N = N(N-1)/2$  qubits, and the energy  $E$  is measured at the end of algorithm execution. In the *adiabatic limit* the result will be  $E = 0$  since  $N < R(m, n)$ . The value of  $N$  is now incremented  $N \rightarrow N+1$ , the AQE algorithm is re-run on  $L_{N+1}$  qubits, and the energy  $E$  measured at the end of algorithm execution. This process is repeated until  $E > 0$  first occurs, at which point the associated  $N$  will be equal to  $R(m, n)$ . Note that any real application of AQE will only be approximately adiabatic. Thus the probability that the measured energy  $E$  will be the ground-state energy will be  $1 - \epsilon$ . In this case, the algorithm must be run  $k \sim \mathcal{O}(\ln[1 - \delta]/\ln \epsilon)$  times so that, with probability  $\delta > 1 - \epsilon$ , at least one of the measurement outcomes will be the true ground-state energy. We can make  $\delta$  arbitrarily close to 1 by choosing  $k$  sufficiently large.

**Simulation Results:** To test the adiabatic quantum computation of  $R(m, n)$ , we numerically simulated the Schrodinger dynamics generated by the AQE Hamiltonian  $H(t)$ . Clearly, these simulations can only be run at finite values of  $T$ . As in Ref. [10], we chose  $T$  so that the algorithm success probability  $P_s$  is large compared to the probability that a randomly chosen CBS will belong to the  $D$ -degenerate ground-state eigenspace of  $H_P$  ( $P_s \gg D/2^L$ ). Here  $P_s$  is the probability that an energy measurement done at the final time  $T$  will yield the ground-state energy  $E_{gs}$  of  $H_P$ . Since a classical computer cannot efficiently simulate the dynamics of a quantum system, we can only obtain small Ramsey numbers. In this case,  $H_P$  can be found by evaluating the cost function  $h(G)$  using the procedure described above Eq. (2).

We simulated the AQE computation of  $R(3, 3)$  and  $R(2, s)$  for  $5 \leq s \leq 7$ . Straightforward arguments [3] give

$R(3, 3) = 6$  and  $R(2, s) = s$ . We present our simulation results in Table I. We see that for all  $m, n$  considered, the threshold value  $N_t$  where  $E_{gs} > 0$  first occurs is precisely at the Ramsey number:  $N_t = R(m, n)$ .

For  $R(2, s)$  and  $N = s$ , Table I gives  $E_{gs} = 1$ . For these cases, graphs corresponding to ground-states of  $H_P$  will thus contain either a single  $s$ -independent set or a single 2-clique. There is only one  $s$ -vertex graph with an  $s$ -independent set, and there are  $C(s, 2) = s(s-1)/2$  graphs with one 2-clique (viz. edge). Thus the ground-state degeneracy  $D = 1 + C(s, 2)$ , in agreement with the  $R(2, s)$  degeneracies in Table I for  $N = s = 5, 6, 7$ . For  $R(3, 3)$  and  $N = 6$ , Table I gives  $E_{gs} = 2$ . Thus graphs corresponding to ground-states are those with: (i) two 3-cliques; (ii) two 3-independent sets; or (iii) one 3-clique and one 3-independent set. Ref. [11] derived the minimum number of 3-cliques and 3-independent sets that can be present in an  $N$ -vertex graph. This minimum is precisely our  $E_{gs}$  for  $R(3, 3)$  and a given  $N$ . For  $N = 6$ , the minimum value is 2, in agreement with  $E_{gs} = 2$  in Table I. We carried out both analytical [12] and numerical counts of the ground-state graphs for  $R(3, 3)$  and  $N = 6$ . Both approaches found 1760 graphs giving a ground-state degeneracy  $D = 1760$ . In all cases appearing in Table I, the upward jump in  $D$  seen upon reaching the Ramsey threshold  $N = R(m, n)$  (from below) is responsible for the jump in the success probability  $P_s$  also seen at this threshold.

Although we would like to have calculated larger Ramsey numbers, this was simply not practical. Note that the  $N = 7$  simulations use  $L = 21$  qubits. These simulations are at the upper limit of 20-22 qubits at which simulation of the full AQE Schrodinger dynamics is practical [10, 13, 14]. The next smallest Ramsey number is  $R(2, 8) = 8$  which requires a 28 qubit simulation, well beyond what can be done practically.

**Experimental Implementation:** We begin by determining an operator expression for the problem Hamiltonian  $H_P$  which then fixes the AQE Hamiltonian  $H(t)$  through Eqs. (3) and (5). Recall that the eigenvalue  $h(G) = \mathcal{C}(G) + \mathcal{I}(G)$  counts the total number of  $m$ -cliques and  $n$ -independent sets in a graph  $G$ . For an  $m$ -vertex set  $S_\alpha = \{v_1, \dots, v_m\}$ , we define the edge set  $E_\alpha = \{e_k^\alpha : k = 1, \dots, C(m, 2)\}$  as the set of all edges connecting pairs of vertices  $v_i, v_j \in S_\alpha$ , and  $C(m, 2)$  is the number of ways of choosing 2 vertices out of  $m$ . If  $S_\alpha$  corresponds to an  $m$ -clique in the graph  $G$ , the graph-string  $g(G)$  must have 1's at all bit-positions associated with the edges of  $E_\alpha$ . Let the states  $|0\rangle$  and  $|1\rangle$  satisfy  $\sigma_z|a\rangle = (-1)^a|a\rangle$ . Then the operator  $H_\alpha = \prod_{e \in E_\alpha} P_1^e$  (where  $P_1^e = (1/2)[I^e - \sigma_z^e]$ , and  $e$  labels the qubit associated with edge  $e$ ) will have  $|g(G)\rangle$  as an eigenstate with eigenvalue 1 when  $S_\alpha$  is an  $m$ -clique, and zero otherwise. The operator that counts all  $m$ -cliques in a graph  $G$  is then  $H_{cl}^m = \sum_{\alpha=1}^{C(N, m)} H_\alpha$ , and by construction,  $H_{cl}^m|g(G)\rangle = \mathcal{C}(G)|g(G)\rangle$ . A similar analysis can be carried out for  $n$ -

independent sets. Let  $T_\alpha = \{v_1, \dots, v_n\}$  be an arbitrary  $n$ -vertex set, and  $\bar{E}_\alpha$  its corresponding edge set. If  $T_\alpha$  is an  $n$ -independent set in a graph  $G$ , then the graph-string  $g(G)$  must have 0's at all bit-positions associated with the edges of  $\bar{E}_\alpha$ . The operator  $\bar{H}_\alpha = \prod_{e \in \bar{E}_\alpha} P_0^e$  (where  $P_0^e = (1/2)[I^e + \sigma_z^e]$ , and  $e$  labels the qubit associated with edge  $e$ ) will have eigenstate  $|g(G)\rangle$  with eigenvalue 1 (0) when  $T_\alpha$  is (is not) an  $n$ -independent set. The operator that counts all  $n$ -independent sets in an arbitrary graph  $G$  is then  $H_{is}^n = \sum_{\alpha=1}^{C(N, n)} \bar{H}_\alpha$ , and by construction,  $H_{is}^n|g(G)\rangle = \mathcal{I}(G)|g(G)\rangle$ . For calculation of  $R(m, n)$ , the problem Hamiltonian  $H_P^{Nmn}$  is then

$$H_P^{Nmn} = H_{cl}^m + H_{is}^n. \quad (6)$$

Note that  $H_P^{Nmn}$  contains  $\mathcal{O}(N^s)$  terms, where  $N$  is the number of vertices and  $s = \max\{C(N, m), C(N, n)\}$ . Since each  $H_\alpha$  and  $\bar{H}_\alpha$  is a projection operator, their operator norm will be unity and their matrix elements, being 0's and 1's, are specified with a single bit. Lastly, note that each term in  $H_P^{Nmn}$  is a product of at most  $t = \max\{C(m, 2), C(n, 2)\}$   $\sigma_z$ -operators so that  $H_P^{Nmn}$  is a  $t$ -local Hamiltonian [15]. By using perturbative gadgets, it can be reduced to a 2-local Hamiltonian [16–18].

For a given Hamiltonian  $H(t)$ , two approaches have been demonstrated to experimentally implement AQE [19–21]. Refs. [19], [20] partitioned the full evolution into  $\mathcal{N}$  subintervals of duration  $\Delta t = T/\mathcal{N}$  which are sufficiently short that the propagator  $U_l$  for each subinterval  $l$  can be factored via a Trotter expansion. This approach was applied to three-qubit systems, though it can be used for arbitrary size qubit systems. Ref. [21] describes experiments using a quantum annealing device designed to implement adiabatic quantum optimization algorithms. Results are reported of AQE solution for the groundstate of randomly generated instances of an 8-qubit quantum Ising spin glass. Work using perturbative gadgets is underway to convert  $H_P^{Nmn}$  into a 2-local form amenable to both AQE experimental approaches.

**Ramsey Numbers and QMA:** Quantum complexity theory formalizes the notion of efficient quantum algorithms. Our interest is in the quantum complexity class *QMA* which generalizes the randomized version of the classical complexity class *NP* [15, 16].

*QMA* is a class of promise problems where each problem  $L$  is the union of two disjoint sets of binary strings  $L_y$  and  $L_n$  corresponding to Yes and No instances of the problem. For a string  $x \in L_y \cup L_n$ , the task is to determine whether  $x \in L_y$  or  $x \in L_n$  using polynomial resources. Let  $\mathcal{H}$  denote a two-dimensional Hilbert space; and  $|x\rangle$  the CBS labeled by the binary string  $x$ .

**Definition 1 (QMA)** Let  $x \in L = L_y \cup L_n$  and  $\epsilon = 2^{-\Omega(|x|)}$ . The promise problem  $L$  belongs to *QMA* if there exists a quantum polynomial-time verifier  $V(|x\rangle, |y\rangle) \rightarrow \{0, 1\}$ , and a polynomial  $\pi(|x|)$  such that: (i) for all  $x \in L_y$ , there exists an  $|\xi\rangle \in \mathcal{H}^{\pi(|x|)}$

TABLE I: Simulation results for Ramsey numbers  $R(3, 3)$  and  $R(2, s)$  for  $5 \leq s \leq 7$ . Here  $N$  is the number of graph vertices;  $E_{gs}$  and  $D$  are the ground-state energy and degeneracy, respectively, for the problem Hamiltonian  $H_P$ ; and  $T$  and  $P_s$  are, respectively, the algorithm runtime and success probability.

R(2, 5)					R(2, 6)					R(3, 3)					R(2, 7)				
$N$	$E_{gs}$	$D$	$T$	$P_s$	$N$	$E_{gs}$	$D$	$T$	$P_s$	$N$	$E_{gs}$	$D$	$T$	$P_s$	$N$	$E_{gs}$	$D$	$T$	$P_s$
3	0.0	1	5.0	0.591	4	0.0	1	5.0	0.349	4	0.0	18	5.0	0.769	5	0.0	1	8.0	0.865
4	0.0	1	5.0	0.349	5	0.0	1	5.0	0.173	5	0.0	12	5.0	0.194	6	0.0	1	8.0	0.805
5	1.0	11	5.0	0.518	6	1.0	16	5.0	0.286	6	2.0	1760	5.0	0.693	7	1.0	22	8.0	0.938

such that  $\Pr\{V(|x\rangle, |\xi\rangle) = 1\} \geq 1 - \epsilon$ ; and (ii) for all  $x \in L_n$  and  $|\xi\rangle \in \mathcal{H}^{\pi(|x|)}$ ,  $\Pr\{V(|x\rangle, |\xi\rangle) = 1\} \leq \epsilon$ . Here  $\Pr\{V(|x\rangle, |\xi\rangle) = 1\}$  is the probability that  $V$  concludes  $x \in L_y$  when the quantum witness is  $|\xi\rangle$ .

Informally, if  $x$  is a Yes (No) instance, there exists a (no) quantum witness  $|\xi\rangle$  which causes  $V$  to correctly (mistakenly) conclude  $x \in L_y$  with probability at least  $1 - \epsilon$  (greater than  $\epsilon$ ).

A promise problem is *QMA-Complete* if it belongs to *QMA* and all problems in *QMA* are polynomially reducible to it. It has been shown [15, 16] that *k-Local Hamiltonian* is *QMA-Complete* for  $k \geq 2$ .

**Definition 2 (*k-Local Hamiltonian*)** Consider an  $L$ -qubit Hamiltonian  $H = \sum_{j=1}^r H_j$ , where  $r = \text{poly}(L)$ ; and each term  $H_j$  acts on at most  $k$  qubits (*k-local*); has operator norm  $\|H_j\| \leq \text{poly}(L)$ ; and matrix elements specified by  $\text{poly}(L)$  bits. Finally, two constants  $a < b$  are specified. The Hamiltonian  $H$  is a Yes instance if its groundstate energy  $E_{gs} < a$ , and a No instance if  $E_{gs} > b$ . The problem is, given a *k-local Hamiltonian*  $H$ , determine whether  $H$  is a Yes or a No instance.

Our Ramsey number AQE algorithm leads naturally to an example of *t-Local Hamiltonian* which we call *RAMSEY*. We have seen that the Ramsey problem Hamiltonian  $H_P^{Nmn}$  is a *t-local Hamiltonian*; is a sum of a polynomial number of terms  $H_j = H_\alpha$  or  $\bar{H}_\alpha$ ; and each  $H_j$  satisfies the polynomial bounds specified in Definition 2. Suitable choices for the constants  $a$  and  $b$  are  $0.01 < a < 0.1$  and  $b = 1 - a$ . Yes instances of *RAMSEY* then correspond to  $N < R(m, n)$  since  $E_{gs} = 0 < a$ , and No instances to  $N \geq R(m, n)$  where  $E_{gs} \geq 1 > b$ . It is possible to carry over the proof that *k-Local Hamiltonian* is in *QMA* [15] to show that *RAMSEY* is also in *QMA*.

For an AQE algorithm with *non-degenerate* ground-state (GS), the runtime is largely determined [8] by the minimum energy gap  $\Delta = \min_t \{E_1(t) - E_0(t)\}$ . This connection fails for the Ramsey algorithm when  $N = R(m, n)$  as the GS becomes *degenerate* during its execution and so  $\Delta$  vanishes. Determining how the runtime scales when  $\Delta = 0$  (as with the Ramsey algorithm) is an open problem in adiabatic quantum computing.

In this paper we have presented a quantum algorithm that calculates two-color Ramsey numbers  $R(m, n)$ ; nu-

merically simulated the algorithm and shown it correctly determined small Ramsey numbers; discussed its experimental implementation; and shown that Ramsey number computation is in the quantum complexity class *QMA*.

We thank W. G. Macready, P. Young, S. Jordan, and M. J. O'Hara for valuable comments, and F. G. thanks T. Howell III for continued support.

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